FRICTION OF FLUID FLOW AGAINST THE END-FACE SURFACES OF VORTEX CHAMBERS

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In order to estimate momentum losses in vortex chambers, it is necessary to have information about the turbulent friction of the rotating flow against the surface perpendicular to the axis of rotation. In hydraulic calculations, a quadratic friction law is often used:

$$\tau = \zeta \rho u^2/2, \tag{1}$$

where τ is the tangential stress at the wall; ρ, density of the fluid; u, relative velocity of the flow around the surface; and ζ, coefficient of friction. Recommendations for the choice of the latter are very contradictory and do not directly concern the object of our investigation. For this reason, the purpose of this paper is to check relation (1) experimentally and to determine the coefficient ζ.

Experimental Setup. The scheme of the experimental setup is shown in Fig. 1. Its basic element is a vortex chamber 1 with an inner diameter 350 mm and height 32 mm. Water flows into the chamber through a slotted directing apparatus 2, which contains N millimeter slits, inclined to the radius at an angle $\alpha = 60°$. The number N was varied from 18 to 87. The velocity of the water in the slits was varied from 1 to 15 m/sec, which covers the range of practical interest.

The friction of the flow against the wall is measured with a thin flat disk 3, fixed with the help of spokes onto the shaft 4, which can rotate freely on bearings placed in the upper end face of the top. With the help of a friction break 5, a variable braking torque, measured with the help of a dynamometer 6, can be applied to the shaft. The following quantities are measured in the experiment: the fluid velocity in the slit v_s, the angular rotational velocity of the disk ω, and the braking torque M. Preliminary experiments were conducted to find the law governing the circular velocity distribution of the fluid in the chamber which turned out to be very close to a potential rotation

$$v_\varphi = R_0 v_{s1} \sin \alpha / r, \tag{2}$$

where R_0 is the radius of the chamber and r is the running radius. We note that an air cavity with radius $R_a < R_1$ always appeared at the center of the flow.

Friction Law. In the case of friction of a fluid against a rotating disk, relation (1) takes the form

$$\tau = (1/2)\zeta\rho |v_\varphi - \omega r|(v_\varphi - \omega r). \tag{3}$$

Here the modulus of the relative velocity is separated in connection with the fact that it can change sign and the quantity τ is assumed to be positive if the flow overtakes the disk.

The moment of the friction forces is calculated using the equation

$$M = 2 \cdot 2\pi \int_{R_1}^{R_2} r^2 \tau dr. \tag{4}$$

Substituting expressions (3) and (2) into (4) and assuming that ζ = const, we obtain the relation

$$M = (2/15)\pi\rho\zeta v^2 R^3 [16y^{-1/2} - 15(1+\xi) + 10(1+\xi^3)y - 3(1+\xi^5)y^2], \quad y > 1; \tag{5}$$

$$M = 2\pi\rho\zeta v^2 R^3 [(1-\xi) - (2/3)(1-\xi^3)y + (1/5)(1-\xi^5)y^2], \quad y < 1, \tag{6}$$

where $v = v_\varphi(R) = v_{s1} \sin \alpha \cdot R_0/R$; $\xi = R_1/R_0$; and $y = \omega R/v$. In the case $y < 1$, the disk rotates more slowly than the flow, so that this case also includes a disk at rest. In the case $y > 1$, the external layers of the disk

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TABLE 1

ξ	0,5	0,5	0,5	0,5	0,5	0,5	0,75	0,75	0,75	0,75	0,75
v, m/sec	1,3	7,5	15	7,5	15	5,2	1,2	4,6	9,0	15	15
M, N·m	0,11	3,47	14,2	3,3	10,4	0,76	0,045	0,65	2,7	7,4	3,8
y	0	0	0	0,12	0,28	0,6	0	0	0	0	0,36
$\zeta \cdot 10^{-3}$	5,3	4,8	4,9	5,2	5,1	5,0	4,9	4,8	5,2	5,1	5,0

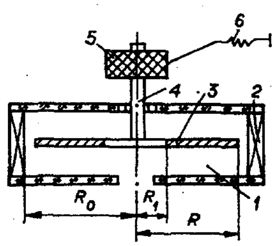

Fig. 1

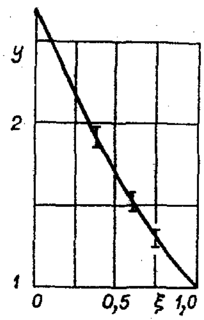

Fig. 2

have a local velocity exceeding v_φ, while the inner layers have a local velocity that is less than v_φ. The freely rotating disk, for which M = 0, relates to the case $y > 1$. As can be seen from (5), for M = 0, the quantity y is uniquely related to the geometric parameter ξ and does not depend either on the flow regime v or on the coefficient of friction ζ. This property can be used to check the friction law (3) experimentally.

The characteristic $y(\xi)$, constructed based on (5) with M = 0, is shown in Fig. 2. The experimental data, relating to different flow rates of water, different directing apparatus, and $\xi = 0.375$, 0.5, and 0.75, corresponding with fixed R = 0.16 m to the values $R_1 = 0.06$, 0.08, and 0.12 m, are also shown in this figure. The experimental data fall quite closely, but with some spread, on the curve, so that Fig. 1 shows not the experimental points themselves, but the limits of the spread. The results obtained support quite convincingly the friction law (3), although it is evident that they are somewhat high on the average.

Coefficient of Friction. It is convenient to determine the quantity ζ for a disk at rest or strongly braked, i.e., based on Eq. (6), from which the following working relation is obtained:

$$\zeta = M/\{2\pi\rho v^2 R^3[(1-\xi)-(2/3)(1-\xi^3)y+(1/5)(1-\xi^5)y^2]\}.$$

Selected results obtained by analyzing the measurements are presented in Table 1, which gives an idea of the average value and spread of this quantity. An analysis of a large amount of experimental data permits recommending the value $\zeta = (5 \pm 0.3) \cdot 10^{-3}$.

We note that the quantity $\zeta = 5 \cdot 10^{-3}$ in Eq. (1) was recommended in [1] as a rough approximation even for friction of gas-fluid mixtures against a solid wall, but only with an appropriate choice of density of the mixture.

LITERATURE CITED

1. G. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill (1969).